DYNAMIC HYPERPATHS: THE STOP MODEL

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Abstract

The purpose of this paper is to investigate the possibility of exploiting the hyperpath paradigm to model the dynamic transit assignment in metropolitan networks affected by heavy congestion. The core problem is to develop a stop model for reproducing what happens in the case of passenger queues and to verify which boarding rule is suitable to represent this congestion phenomenon. Different models are proposed depending on the layout of the stop and on the congestion level.

1 Introduction

The concept of *optimal strategy* was firstly introduced by Spiess and Florian (1989) to model the travel behaviour of rational passengers in presence of perceived uncertainty on vehicle arrivals when several routes are available to reach the destination from a transit stop, including the relevant case of *common lines* (i.e. partly overlapping). The strategy is chosen before the beginning of the trip and, starting from the origin, involves the iterative sequence of: walking to a transit stop or to the destination, selecting the *attractive* lines to board and, for each of them, the stop where to alight. According to Nguyen and Pallottino (1988), a transit assignment reproducing this strategic behaviour can be modelled by loading a shortest (i.e. with minimal cost) *hyperpath* that connects the origin of the trip to the destination having the diversion nodes at stops through waiting *hyperarcs*, each of which identifies a line set. Traditionally, hyperpaths have been then exploited to model the frequency-based transit assignment in a static framework, where it is assumed that a passenger, after reaching a stop, waits for the first attractive carrier among a fixed set of lines. It is known (Billi et al., 2004; Noekel, 2007) that this behaviour is rational only when:

- no information is provided at the stop on actual waiting times and on the available capacities of arriving carriers;
- the vehicle arrivals of different lines at the stop are statistically independent, and the same is true for the passenger arrivals with respect to vehicle arrivals;
- the headway probability distribution between two successive vehicles of the same line and hence the waiting time for a passenger randomly arriving at the stop are exponential, i.e. memoryless.

The challenge here is to investigate the possibility of developing a new model for dynamic transit assignment by extending the *hyperpath* paradigm to the case where link travel times and transit frequencies vary during the day. To this end, we need to specify three main components:

- the *route choice model*, which reproduces the behaviour of a passenger travelling from an origin toward a destination on the transit network, for given service performances, including the line boarding probabilities of each hyperarc and its expected waiting time;
- the *Dynamic Network Loading* (DNL), which is aimed at finding time-varying link flows and service performances that are consistent with line capacities, for given route choices expressed in terms of node splitting rates;
- the *stop model*, which yields the probability of boarding each line of any hyperarc and its expected waiting time, for given passenger queues, line frequencies and expected travel times to reach the destination once on-board.

For the route choice model, we could employ a deterministic approach by extending the method proposed by Chabini (1998) to find the dynamic shortest hyperpaths. In addition, we could exploit the General Link Transmission Model (Gentile, 2008) to perform the Dynamic Network Loading. However, these two components are out of the scope of this work, which is dedicated solely to the stop model.

Adopting the approach proposed by Meschini et al. (2007), transit frequencies are conceived here as a continuous flow of line carriers. This allows representing explicit capacity constraints on line vehicles and reproducing "over saturation" queuing times of passengers at transit stops. On the other hand, we have to force into the model the simulation of the "under saturation" waiting time of passengers at transit stops, due to the intrinsic discontinuity of the service, by associating a delay to hyperarcs. Therefore, the core-problem in developing a dynamic transit assignment based on hyperpaths is to define the stop model, investigating what happens in the case of heavy congestion with formation of passenger queues, and to verify which boarding rule is suitable to represent this congestion phenomenon.

We will focus in the following on a single stop and a specific set of attractive lines. Our stop models will allow us to evaluate the expected waiting time of the corresponding hyperarc, as well as the probability to board each attractive line, addressing the classical case of exponential headways. In this situation, it is convenient to board the first attractive carrier that arrives the stop, instead of keep waiting (Gentile *et al.*, 2005), if there is no queue.

Indeed, if the flow willing to board is always under the capacity available on the approaching line vehicle, there is no formation and dispersion of residual queues; passengers can board immediately and experience only the under saturation delay. On the contrary case, passengers have to queue until the service becomes actually available to them, thus suffering an additional over saturation delay. Therefore, the model has to be adjusted to represent the queue dynamic, which depends also on the layout of the stop and on the information eventually provided to passengers.

2 Hyperpath formulation and notation

The transit network is here formally represented by an oriented *hypergraph* $G = (N, A)$, where N is the set of nodes and A is set of arcs and hyperarcs.

As usual, the generic arc $a \subseteq A$ is identified by an ordered pair of nodes, referred to respectively as the *tail*, denoted by $TL(a) \in N$, and the *head*, denoted by $HD(a) \in N$; that is $a = (TL(a), HD(a))$. While for the generic hyperarc the head can be a set of nodes, i.e. $HD(a) \subseteq N$.

We distinguish the following different types of arcs and hyperarcs:

- AP *pedestrian arcs*, used by passengers to walk from the origin to a boarding stop, from a stop to another stop, and from the alighting stop to the destination;
- AH *waiting hyperarcs*, used to model the under saturation delay due to the discontinuity of the service. Passengers at the stop do not know which attractive carrier will arrive first, therefore they associate a probability to each head of the hyperarc that represents the boarding on a particular line;
- AQ *queuing arcs*, used to model the over saturation delay due to passenger flow exceeding the available capacity of the line at the stop;
- AA *alighting arcs*, used to model the alighting process;
- AL *line arcs*, connecting two subsequent stops of a same line;

Therefore we have: $A = AP \cup AH \cup AQ \cup AA \cup AL$.

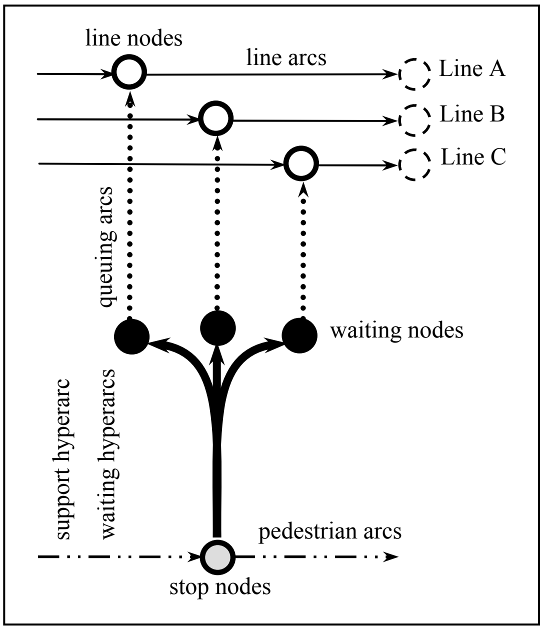

Figure 1 - Representation of a stop in the hypergraph.

Note that the assumption of representing first the waiting process and then the queuing process as in Figure 1 is questionable from a phenomenal point of view, since exactly the opposite occurs in reality; however this proves to be a valid choice from a modeling point of view. First of all, doing so allows us to develop a model with separable queues. On the contrary, if queues were not separate, the model should also represent overtakings among passengers wanting to board different attractive lines and the FIFO rule would not hold true. Secondly, both the expected waiting time and the expected travel time once boarded affect passenger choices as part of the generalized travel costs. Instead, representing the waiting process after the queuing process impedes to include the queuing time in the computation of the optimal strategy. Finally, because we conceive transit frequencies as a continuous flow of line carriers (Meschini et al., 2007), we have to force into the model the representation of the delay due to an actually discontinuous service. Under this consideration, we can add the “under saturation” waiting time at transit stops, wherever it is more comfortable from a modelling point of view, in this case associating it to hyperarcs before the queuing time, as far as all components of generalized costs are correctly taken into account.

The *support hyperarc a* is associated with all the lines serving the stop. However, a passenger directed to a given destination *d* will consider only a subset of services. Therefore, we associate a specific hyperarc *b* to each possible attractive line set, i.e. $HD(b) \subseteq HD(a)$, as depicted in Figure 2.

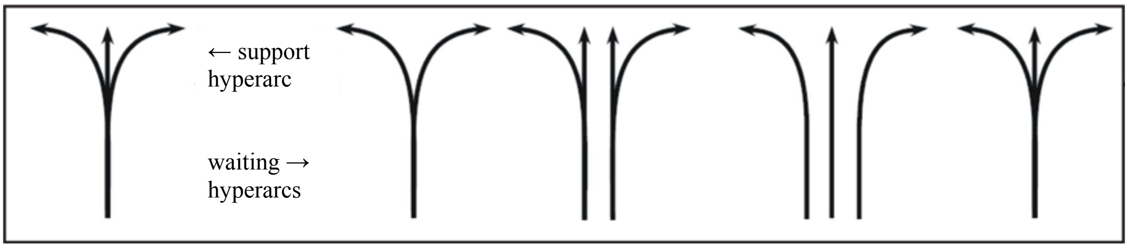

Figure 2 - Wainting hyperarcs defining the attractive set associated to the support hyperarc of all available lines

A hyperpath *h* is an acyclic sub-graph on *G* connecting a single origin *o* to a single destination *d*, where there is one single arc or hyperarc exiting from each node (except for the destination) and all nodes are connected to *d*; that is, diversions occur only at waiting hyperarcs.

In the following we introduce the variables utilized to describe the stop model.

$\lambda_\ell(t)$ frequency of line ℓ departing from the terminal at time t

$T_\ell^i(t)$	instant when the carrier of line ℓ departed from the terminal at time t reaches stop i
$\varphi_\ell^i(t)$	frequency of line ℓ at stop i at time t; it is the inverse of the headway expected value
$Q_\ell^i(t)$	available capacity on line ℓ at stop i at time t
$N_\ell^i(t)$	number of passengers waiting in a queue to access line ℓ at stop i at time t (namely, number of passengers exceeding the available capacity of the approaching carrier)
$M_\ell^i(t)$	number of passengers waiting for service at stop i at time t that are able to board the next carrier of line ℓ
$W_L^i(t)$	expected waiting time of the hyperarc identified by the set of attractive lines L serving stop i at time t
$\pi_{\ell\in L}^i(t)$	probability of boarding line ℓ among the attractive set L at stop i at time t ; equal to the internal coefficients of the corresponding hyperarc
$e_\ell^i(t)$	flow of passengers on line ℓ approaching stop i at time t
Φ_ℓ	vehicle capacity of line ℓ
$P_\ell^i(t)$	probability to be able of boarding the next vehicle of line ℓ approaching stop i at time t
$\psi_\ell^i(t)$	effective frequency of line ℓ at stop i at time t

The temporal profile $\varphi_\ell^i(t)$ of the frequency at a given stop can be determined on the basis of the temporal profile $\lambda_\ell(t)$ of the frequency departing from the terminal and of the travel times on the network by applying a basic dynamic formula:

$$\varphi_\ell^i(T_\ell^i(t)) = \lambda_\ell(t) / (\mathrm{d}T_\ell^i(t) / \mathrm{d}t) \qquad (1)$$

The available capacity is then given by:

$$Q_\ell^i(t) = \Phi_\ell \cdot \varphi_\ell^i(t) - e_\ell^i(t)\,, \qquad (2)$$

Therefore, we have:

$$M_\ell^i(t) = Q_\ell^i(t) / \varphi_\ell^i(t) \qquad (3)$$

Based on the above equations, all variables are time-varying, including the main characteristic of the headway distribution, that is the frequency. However, it is very difficult to consider this feature in the computation of expected waiting time and line probabilities, which require an integration over time. We will henceforth refer to the values of all the variables at the instant when the passengers reaches the stop and consider them to be constant during the wait.

3 The stop model for a single line

3.1 Mingling queue

If the stop is designed as a platform (namely, in the underground case), passengers *mingle* on it and cannot respect any boarding order, as they do not know exactly where the carrier is going to stop. Thus, if a passenger stands just in front of the point where the doors will open, then he will probably board on the next carrier approaching the stop. But if he stands far from that point, he may have to wait for a subsequent arrival. Therefore, the waiting time does not decrease only because a passenger has already missed one or two runs due to congestion. These are the same assumptions made in Schmoecker *et al.* (2008). On such basis, a passenger has the same probability of boarding at each carrier arrival, that can be evaluated as:

$$P_\ell^i(t) = M_\ell^i(t) / (M_\ell^i(t) + N_\ell^i(t)) \qquad (4)$$

Therefore, passengers perceive a service with the following *effective frequency*:

$$\psi_\ell^i(t) = \varphi_\ell^i(t) \cdot P_\ell^i(t) \qquad (5)$$

Given the hypothesis of exponential arrivals with rate $\varphi_\ell^i(t)$, it can be proved (we did it by simulation) that the probability density function of the waiting time for the generic line ℓ is still exponential with a rate equal to the effective frequency:

$f_\ell^i(w, t) = \psi_\ell^i(t) \cdot \exp(-\psi_\ell^i(t) \cdot w)$, if $w \geq 0$; $f_\ell^i(w, t) = 0$, otherwise (6)

and thus the expected waiting time is:

$W_\ell^i(t) = 1 / \psi_\ell^i(t) = 1 / \varphi_\ell^i(t) + N_\ell^i(t) / Q_\ell^i(t)$ (7)

This is consistent with the intuition, that calls for scaling the frequency by the probability to be able of boarding. Moreover the waiting time can be decomposed into a *service time* and a *queuing time*.
Clearly, when the congestion level is extremely high the queue may spill back from the platform and can assume a FIFO behaviour; this phenomenon can be modelled by a link to access the platform with a final bottleneck.

3.2 FIFO queue

If the stop is designed so that passengers have to respect a FIFO service order, then who arrives as first at the stop is the first to board. Hence, the $N_\ell^i(t)$-th queuing passenger will have to wait for the $k_\ell^i(t)$-th arrival, when the service will be truly available to him:

$k_\ell^i(t) = 1 + \mathrm{INT}[N_\ell^i(t) / M_\ell^i(t)]$ (8)

where INT[x] is the first integer not smaller than x.
Given that the headway is exponentially distributed, the arrival of the $k_\ell^i(t)$-th vehicle has an Erlang distribution:

$f_\ell^i(w, t) = \exp(-\varphi_\ell^i(t) \cdot w) \cdot \varphi_\ell^i(t) \wedge k_\ell^i(t) \cdot w \wedge (k_\ell^i(t)-1) / (k_\ell^i(t)-1)!$, if $w \geq 0$; $f_\ell^i(w, t) = 0$, otherwise (9)

Relaxing $k_\ell^i(t)$ to a continuous variable:

$k_\ell^i(t) = 1 + N_\ell^i(t) / M_\ell^i(t) = 1 / P_\ell^i(t)$ (10)

yields the following Gamma distribution of waiting time:

$f_\ell^i(w, t) = \exp(-\varphi_\ell^i(t) \cdot w) \cdot \varphi_\ell^i(t) \wedge k_\ell^i(t) \cdot w \wedge (k_\ell^i(t)-1) / \Gamma(k_\ell^i(t))$, if $w \geq 0$; $f_\ell^i(w, t) = 0$, otherwise (11)

and thus the expected waiting time is:

$W_\ell^i(t) = k_\ell^i(t) / \varphi_\ell^i(t) = 1 / \psi_\ell^i(t) = 1 / \varphi_\ell^i(t) + N_\ell^i(t) / Q_\ell^i(t)$ (12)

Consequently, the expected waiting time at the stop is the same as in the mingling case, given by the service time and a queuing time. The variance of the Erlang or Gamma function is instead lower than the variance of the Exponential function for the same expected value.

4 The stop model for multiple lines

4.1 Mingling queue

Let us consider a stop where passengers *mingle* while waiting to board the first attractive line. When the available capacity of approaching carriers is lower than the number of passengers at the stop willing to board a line, the waiting time has an exponential distribution with rate equal to the effective frequency computed by (5).
On this basis it is possible to compute as in Gentile *et al.* (2005) the probability of line ℓ to be the first line where the passenger is able to board among the attractive set L, given that the vehicle arrivals of different lines are statistically independent. Therefore, the internal coefficient of the corresponding *hyperarc* is equal to the ratio between the effective frequency of the line ℓ and the sum of the effective frequencies of all the attractive lines $j \in L$:

$\pi_{\ell\in L}^i(t) = \psi_\ell^i(t) / \sum_{j\in L} \psi_j^i(t)$ (13)

while the expected waiting time is the inverse of the cumulated effective frequencies:

$W_L^i(t) = 1 / \sum_{j\in L} \psi_j^i(t)$ (14)

4.2 separate FIFO queue

If the transit system is highly crowded, the stops shared by several lines can be designed to have physically separate queues for the different lines. This situation should be modelled in different ways according to the actual congestion level.
In this case, each line where $N_\ell^i(t) > 0$ cannot be considered for a strategic behaviour, since the passenger has to join the corresponding queue as soon as he reaches the stop and then it may be difficult for him to change row. This case thus reduces to that of a FIFO queue for a single line that we already have examined.
All the other lines can instead be included into non-trivial attractive sets. Since there is no queue congestion, the stop model reduces to the classical exponential case, where:

$$\pi_{\ell\in L}^i(t) = \varphi_\ell^i(t) / \sum_{j\in L} \varphi_j^i(t) \qquad (15)$$

$$W_L^i(t) = 1 / \sum_{j\in L} \varphi_j^i(t) \qquad (16)$$

However, if passengers were provided with information at the stop regarding the arrival times of carriers and the available capacity on-board (or the passenger has sufficient experience to guess it), we could still model passengers' behaviour through hyperpaths also in the case of queues. Indeed, the information anticipates the event of vehicle's arrival to the moment when the passenger reaches the stop; hence, his optimal travel strategy comes true at this instant, when he actually chooses which line to board taking into account the length of the different queues.

4.3 mixed FIFO queue

The last model we discuss represents a stop served by a set of different lines, where passengers wait together in a single queue to board the first carrier of their own attractive set. In this case overtaking is possible among passengers having different attractive set; however, any competition among passengers willing to board an approaching line is solved applying the FIFO rule.
The over saturation queue of each line determines which $k_\ell^i(t)$-th vehicle the passenger is waiting for based on equation (10), while equation (11) expresses the probability density function that the $k_\ell^i(t)$-th arrival occurs after a waiting time w.
On this basis we can proceed as in Gentile et al. (2005). Let $F_\ell^i(w, t)$ be the cumulative distribution of such a waiting time. The probability of boarding line $\ell\in L$ is equal to the probability that its $k_\ell^i(t)$-th arrival occurs before that of all the other lines:

$$\pi_{\ell\in L}^i(t) = \int_0^\infty \gamma_{\ell\in L}^i(w, t) \cdot dw \qquad (17)$$

$$\gamma_{\ell\in L}^i(w, t) = f_\ell^i(w, t) \cdot \prod_{j\neq\ell\in L} (1-F_\ell^i(w, t)) \qquad (18)$$

Finally, we can obtain the expected waiting time as:

$$W_L^i(t) = \sum_{\ell\in L} \int_0^\infty \gamma_{\ell\in L}^i(w, t) \cdot w \cdot dw \qquad (19)$$

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